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by

D.D. Barbosa

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RESISTIVE DECAY OF PHOTOSPHERIC CURRENT SHEETS

by

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ABSTRACT

We consider the magnetostatics of thin, axi-symmetric current sheets in the plane of the photosphere. The resistive decay time constant for energy and flux of the system is computed and found to depend linearly on the thickness of the current sheet.

I. INTRODUCTION

The possibility that photospheric neutral winds can blow across solar magnetic fields and thereby drive photospheric and coronal electric currents has been considered by Sen and White (1972) and Heyvaerts (1974) [see also Obayashi (1975)]. The primary motivation of these authors for developing a dynamo theory was to model a current system which, after the onset of an instability, could be rapidly dissipated and provide sufficient energy for a flare. More recently, Barbosa (1978) has considered some of the more general aspects of photospheric dynamo action.

One important conclusion shared by all of these authors is that when a photospheric current system is confined to a thin sheet the thickness of the sheet plays a central role in almost every aspect. In particular, the resistive time constant of the system should depend on the thickness. The question is, just how?

In Section II we consider the magnetostatics of axi-symmetric, azimuthal current sheets and describe one simple case for illustrative purposes. In Section III we compute the resistive time constants (energy and flux) for these systems.

II. MAGNETOSTATICS

We consider current restricted to flow in a thin sheet at $z=0$ $[(\rho,\phi,z)$ coordinates$]$ where the thickness and perpendicular scale length satisfy $\ell \ll r$. The current density is modelled as

$$\vec{J}(\vec{x}) = K(\rho)\,\frac{1}{\ell}\left[H(z+\ell/2) - H(z-\ell/2)\right]\hat{\phi} \simeq K(\rho)\,\delta(z)\,\hat{\phi}\,, \qquad (1)$$

where $H(z)$ is the Heaviside step function and $K(\rho)$ is the surface current density in the sheet with scale length r. The magnetic vector potential for this system is given as [Jackson, 1962]

$$A_\phi(\vec{x}) = \frac{2\pi}{c}\int_0^\infty \rho'\, d\rho'\; K(\rho')\int_0^\infty dk\; J_1(k\rho)J_1(k\rho')\; e^{-k|z|} \qquad (2)$$

and the field lines may be obtained from the Euler potential

$$\alpha = \rho A_\phi = \text{constant}. \qquad (3)$$

The magnetic field in cylindrical coordinates is

$$B_z = \frac{1}{\rho}\frac{\partial\alpha}{\partial\rho}\,, \qquad B_\rho = -\frac{1}{\rho}\frac{\partial\alpha}{\partial z}\,. \qquad (4)$$

We shall make use of the Fourier-Bessel transformation throughout this paper

$$\left.\begin{aligned} \tilde{K}(k) &= \int_0^\infty \rho\, d\rho\; K(\rho)J_1(k\rho) \\ K(\rho) &= \int_0^\infty k\, dk\; \tilde{K}(k)J_1(k\rho) \end{aligned}\right\}. \qquad (5)$$

Thus, the vector potential is simply

$$A_\phi = \frac{2\pi}{c} \int_0^\infty dk\, \tilde{K}(k) J_1(k\rho) e^{-k|z|} \quad , \tag{6}$$

and the total current flowing is

$$I = \int_0^\infty d\rho\ K(\rho) = \int_0^\infty dk\ \tilde{K}(k) \ . \tag{7}$$

Relatively simple expressions for the magnetic field may be obtained from the class of current densities given by

$$K(\rho) = K_0 \frac{r^{2n}\rho}{(\rho^2+r^2)^{n+1/2}} \ . \tag{8}$$

Gleeson and Axford (1976) have considered this class of currents in connection with current sheets in the Jovian magnetosphere. We do not propose that they are accurate representations of a photospheric current system. They serve only as convenient models which possess easily describable features and for which our calculations may be performed with significant tractability.

The case of $n=1$ is a monopolar field with straight field lines which behaves as $B \sim \rho^{-2}$. A more realistic case is given by $n=2$ for which the field lines are derived from

$$\alpha = \frac{2\pi K_0 r^2}{3c} \frac{r\rho^2}{[\rho^2+(|z|+r)^2]^{3/2}} = \frac{2\pi K_0 r^2}{3c}\ \bar{\alpha} \ . \tag{9}$$

This configuration has an O-type neutral point at $\rho = \sqrt{2}\ r$ and the current links a maximum (normalized) flux there of $\bar{\alpha}_{MAX} = 2/\sqrt{27}$.

Representative field lines are shown in Figure 1. The actual magnetic flux distribution is given by

$$\Phi = 2\pi\alpha \ . \tag{10}$$

III. RESISTIVE TIME CONSTANTS

To compute the resistive decay time, we assume that a given current system (1) has been set up and at t=0 all electromotive forces are shut off and the system decays through ohmic losses. Integration over all space yields

$$E = \frac{1}{8\pi}\int d\vec{x}\; B^2 = \frac{1}{2c}\int d\vec{x}\; \vec{J}\cdot\vec{A} \quad , \tag{11}$$

and making use of (5) we find that the total energy of the system is given by

$$E = \frac{2\pi^2}{c^2}\int_0^\infty dk\; \tilde{K}^2(k) \quad . \tag{12}$$

The self-inductance of the system is defined by $E = \frac{1}{2} LI^2$ and therefore

$$L = \frac{4\pi^2}{c^2}\;\frac{1}{I^2}\int_0^\infty dk\; \tilde{K}^2(k) \quad . \tag{13}$$

Ohmic losses occur throughout the volume of the current sheet and Barbosa (1978) has shown that the power dissipated per unit area of the sheet is given as

$$\mathcal{P} = \frac{K^2(\rho)}{\sigma_3 \ell} \quad , \tag{14}$$

where σ_3 is the Cowling conductivity. Integration over the entire area of the sheet gives the total power dissipated and upon equating this to I^2R we find that the effective resistance of the system is given by

$$R = \frac{2\pi}{\sigma_3 \ell} \frac{1}{I^2} \int_0^\infty k \, dk \, \tilde{K}^2(k) \; . \tag{15}$$

Elementary circuit theory gives time constants for decay of the energy and current (or flux) as

$$\tau_E = L/2R \quad , \qquad \tau_\Phi = L/R \; . \tag{16}$$

Thus, the time constant for decay of magnetic flux is (esu)

$$\tau_{DIFF} = \tau_\Phi = \frac{2\pi\sigma_3 \ell r}{c^2} \, g \; , \tag{17}$$

where g is a geometrical factor of order one,

$$g = \frac{1}{r} \frac{\int_0^\infty dk \, \tilde{K}^2(k)}{\int_0^\infty k \, dk \, \tilde{K}^2(k)} \tag{18}$$

The evaluation of g for the model of (8) yields $g = 2, 2/3$ for n=1,2 respectively.

Our basic result is that the decay time depends only linearly on the thickness ℓ for the simple reason that magnetic energy is stored in a volume $\sim r^3$ but ohmic losses occur in a volume $\sim \ell r^2$.

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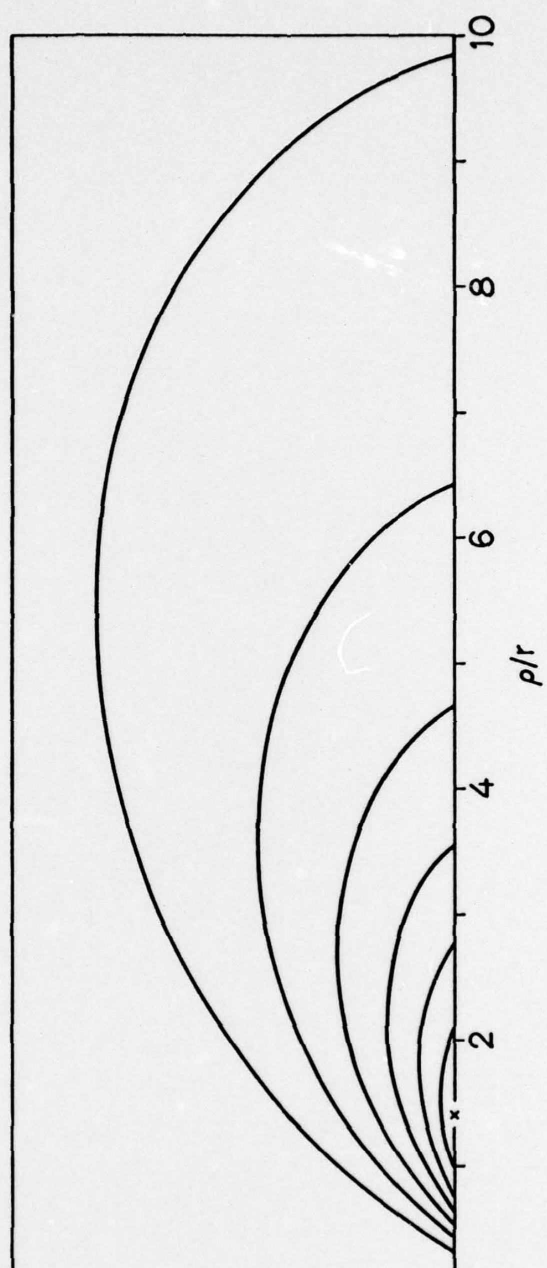
CAPTION FOR FIGURE

Figure 1. Field lines of a current sheet in the $z=0$ plane for $n=2$ in Equation (8). The normalized flux surfaces $\bar{\alpha}$ satisfy Equation (9) and are displayed for $\bar{\alpha} = 0.1$ to 0.35 in steps of 0.05. $\bar{\alpha}_{MAX} \simeq 0.38$ occurs at $\rho = \sqrt{2}\, r$.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